Sample of Material from MAT250 at Broome Community College

This material is for sample purposes only and is not to be considered as an official listing of topics.

1. Consider the product $\prod_{k=1}^{n}\left(1+\frac{1}{k}\right)$. Write out and evaluate this for n = 1, 2, and 3, and conjecture a general formula for this product.

2. For the universal set **R** and subsets $A=\{x\in\mathbf{R}\mid 0<x\leq 2\}$, $B=\{x\in\mathbf{R}\mid 1\leq x\leq 4\}$, and $C=\{x\in\mathbf{R}\mid 3\leq x<9\}$, find $A\cup B$, $A-B$, $B\cap C$, and $A^c\cap C^c$.

3. Two faces of a six-sided die are painted red, two are painted blue, and two are painted green. The die is rolled three times and the face-up color is recorded. Write down the sample space of all possible outcomes of this process. What is the probability of exactly two of the colors matching?

4. Prove that, for all integers $n\geq 1$, $2+4+6+\cdots+2n=n^2+n$.

5. Use an element argument to prove that, for all sets *A*, *B*, and *C*, $A-(A-B)=A\cap B$.

6. Use a Venn diagram to show that the following statement is false, then construct a counterexample: For all sets *A*, *B*, and *C*, $A-(B-C)=(A-B)-C$.

7. A group of eight people are going to a movie. Two of them insist that they must sit next to each other, and two others insist that they do not want to sit next to each other. In how many ways can these eight people be arranged in one row so that both pairs of people are satisfied? (Assume there are no empty seats between them)

8. A large pile of coins consists of pennies, nickels, dimes, and quarters (at least 30 of each). What is the probability that a collection of 30 coins chosen at random will contain at least 4 of each type?

9. Think of a set with $m+n$ elements as composed of two parts, one with *m* elements and the other with *n* elements. Use a combinatorial argument to show that, for $0\leq r\leq\min(m,n)$,

$$\binom{m+n}{r}=\binom{m}{0}\binom{n}{r}+\binom{m}{1}\binom{n}{r-1}+\cdots+\binom{m}{r}\binom{n}{0}$$

10. Suppose a person offers to play the following game with you: You draw a card from a standard deck of cards. If it is a face card (J, Q, K), they pay you \$3, but if it is anything else, you pay them \$1. Should you agree to play this game? Use your expected gain or loss to justify your answer.

11. A certain company uses two suppliers, A and B, for one of its parts. It is known that 2% of the parts from A are defective, and 5% of the parts from B are defective. In an inventory of 180 parts, 100 came from A while the rest came from B. Create a table similar to ones we used in class, and use it to answer the question: If a randomly chosen part is found to be defective, what is the probability that it came from supplier A?

12. Define a function $g: \mathbf{Z} \rightarrow \mathbf{Z}$ by $g(n) = 4n - 5$. Is this function one-to-one? Is it onto? Justify your answers.

13. Show that within any set of thirteen integers chosen from 2 to 40, there are at least two integers with a common divisor greater than 1.

14. Show that the set of all bit strings (strings of 0s and 1s) is countable.

15. Define a sequence by the recurrence relation $s_k = s_{k-1} + 2k$, for $k > 0$, where $s_0 = 3$. Find an explicit formula for the s_k.

16. Let $A = \{5, 6, 7, 8, 9, 10\}$, and define a binary relation R on A by $xRy \Leftrightarrow 3|(x-y)$, where x and y are elements of A. Draw a directed graph for this relation.

17. Let A be the set of all strings of length two using the characters "0", "1", and "2". Define an equivalence relation S on A by $sSt \Leftrightarrow$ the sum of the characters in s equals the sum of the characters in t. Give the equivalence classes of S, that is, for each equivalence class write the set of elements that belongs to that class.

18. Use the RSA cipher with $p = 5$, $q = 11$, and $e = 7$, together with the standard alphabet encoding A = 1, B = 2, ..., to encrypt your first name.

19. Either draw or prove that no such graph exists that is a simple graph with nine edges and all vertices of degree 3. (Hint: First determine how many vertices it must have.)

20. Draw the undirected graph represented by the matrix below, then using matrix multiplication tell how many walks there are from vertex 1 to vertex 4 of length 3.

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 2 & 1 \\ 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$