Sample of Material from MAT281 at Broome Community College

This material is for sample purposes only and is not to be considered as an official listing of topics.

1. Given the vector $\mathbf{u} = \langle 3, 0, -4 \rangle$ find the direction angles of **u .**

2. Find the standard equation of a sphere whose diameter has endpoints (3,4,5) and (-5, 2, -9).

3. Given that vector $\mathbf{u} = \langle 3, 0, -4 \rangle$ and vector $\mathbf{v} = \langle 2, 1, 8 \rangle$. Find the following:

a) A vector orthogonal to both **u** and **v**   b) $\mathbf{u} \cdot \mathbf{v}$
c) The angle between **u** and **v** (round to the nearest degree)
d) $\left\| \dfrac{\mathbf{u}+\mathbf{v}}{\|\mathbf{u}+\mathbf{v}\|} \right\|$
e) The projection of **u** onto **v.**
f) The vector component of **u** orthogonal to **v.**

4. Find the set of parametric equations for a line parallel to the line $x = 2t + 1$, $y = 4t = 3$, $z = t$ that passes through the point (-1,2,5).

5. Find the standard equation of a plane that passes through the point (2,2,1) and is parallel to the x-y plane. If need be explain your steps.

6. Sketch 3-space surface $z = x^2$.

7. Identify the following quadric surfaces: (choose from ellipsoid, hyperboloid of one sheet, hyperboloid of two sheets, elliptic cone, elliptic paraboloid, hyperbolic paraboloid). Also identify the given trace for each.

a. $z = x^2 + 4y^2$; trace $x = 0$
b. $\dfrac{x^2}{16} + \dfrac{y^2}{25} + \dfrac{z^2}{9} = 1$; trace $z = 0$
c. $\dfrac{x^2}{16} + \dfrac{y^2}{25} - \dfrac{z^2}{9} = 1$; trace $y = 0$

8. Convert $x^2 + y^2 + z^2 = 9$ to cylindrical and spherical coordinates.

9. Given $f(x,y)=\frac{-xy}{x^2+y^2}$:

a. Find the $\lim_{(x,y)\to(0,0)} f(x,y)$ along the path x = y
b. Find the $\lim_{(x,y)\to(0,0)} f(x,y)$ along the path x = 0
c. Use your answers to both parts a and b to determine if the $\lim_{(x,y)\to(0,0)} f(x,y)$ exists. Explain your answer.

10. Given $f(x,y) = x^4 - 4y^2$, find the maximum value of the directional derivative at the point P=(0,2)

11. Given $f(x,y)=\sqrt{x^2+y^2}$:

a. Find f(1,2) correct to four decimal places.
b. Find dz the total differential in terms of x, y, dx and dy.
c. Calculate dz from (1,2) to (1.05,2.01) correct to four decimal places.

12. Find the unit tangent and the unit normal to $\mathbf{r}(t) = \langle \sin t, 2\cos t \rangle$ at $t = \pi/4$.

13. Find the curvature to $\mathbf{r}(t) = \langle \sin 3t, \cos 2t \rangle$ at $t = \pi/2$.

14. Given w = xlny and $x = s + t^2$ and y = st, find $\frac{\partial w}{\partial t}$ only using the chain rule.

15. Find the parametric equations of the tangent line to the curve of intersection of the surfaces $x^2 + y^2 = 5$ and z = y at the point (1,2,2).

16. Examine the function $f(x,y) = x^2 - 3xy - y^2$ for relative extrema and saddle points.

17. Assuming x and y are positive, use Lagrange multipliers to minimize f(x,y) = 2x+y subject to the constraint xy = 32.

18. Find the volume of the solid in the first octant bounded above by $z = 4 - x^2 - y^2$.

19. Find the centroid of the planar region bounded by $y = x^2$ and y = 2x, given the density function $\delta(x,y) = x + 3y$.

20. Find the centroid of the solid bounded above by $x^2 + y^2 + z^2 = 1$ where $x \ge 0$ and $z \ge 0$.

21. Evaluate the line integral $\int_C (x^2+y^2+z^2)\,dS$ over the indicated path C:

$r(t) = \sin t\,\mathbf{i} + \cos t\,\mathbf{j} + 8t\,\mathbf{k}$ where $0 < t < \pi/2$.

22. Use Green's Theorem to evaluate the line integral $\int_C (x^2 - y^2)dx + 2xydy$ where C is $x^2 + y^2 = a^2$.

23. Find the value of the integral $\int_C \mathbf{F} \bullet \mathbf{dr}$ if $F(x,y) = e^x \sin y\,\vec{i} + e^x \cos y\,\vec{j}$ and $\mathbf{r}(t) = t\vec{i} + 3t\vec{j}$ where $0 \leq t < \underline{4}$ by doing the following:

a. Show that **F** is conservative.
b. Find the potential function
c. Use the potential function to evaluate the integral.

24. Find the flux of F through S, $\iint_S \mathbf{F} \bullet \mathbf{N}dS$ where **N** is the upward unit normal vector to S. $\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and S: $z = 9 - x^2 - y^2$ where $z \geq 0$.

25. Find a vector-valued function whose graph is the cylinder $x^2 + y^2 = 16$.

26. Find the rectangular equation for the surface by eliminating the parameters from the vector-valued function. Identify the surface by name.

$$\mathbf{r}(u,v) = \langle 3\cos v\cos u,\ 3\cos v\sin u,\ 5\sin v\rangle$$

27. Find the curl **F** for the vector field at the point (2,-1,3) if $\mathbf{F} = \langle x^2z, 2xz, yz\rangle$.