

Sample of Material from MAT282 at Broome Community College

This material is for sample purposes only and is not to be considered as an official listing of topics.

1. Solve the following differential equations using the method of guessing $y = x^r$.

a) $x^2 y'' - x y' - 15y = 0$

b) $x^2 y'' + x y' + 9y = 0$

2. Solve the following systems of differential equations using eigenvalues and eigenvectors.

a)
$$\begin{aligned} x' &= x + y + 2z \\ y' &= x + 2y + z \\ z' &= 2x + y + z \end{aligned}$$

b)
$$\begin{aligned} x' &= 5x + 9y \\ y' &= -x - y \end{aligned}$$

3. Solve: $y' = 3x^4 + 2y$; $y(0) = 2$ by taking derivatives of the original equation and writing the answer in terms of a power series. Give the first four nonzero terms.

4. Solve: $y' + 3xy = \cos x$ using the method of power series. Write out any terms at the start of the series that cannot be nicely put into the summation, then show the summation form for the rest. ($*** + *** + \dots + \sum_{k=?}^{\infty} (** + \dots + **)x^k$).

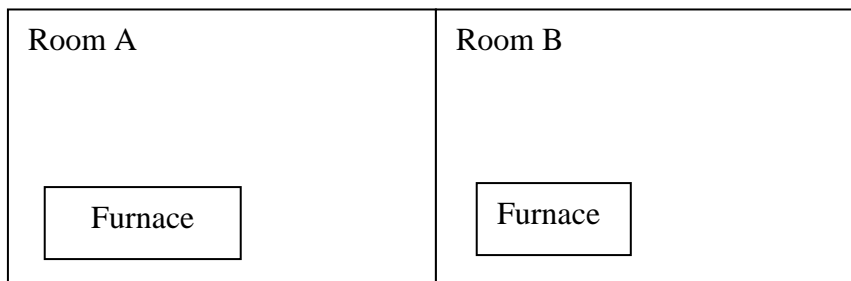
5. Solve $y'' + 7y' + 12y = 13e^{-3t} + 17t$ using variation of parameters.

6. Use the method of elimination to reduce the following systems of equations to a second order linear differential equation with $y(t)$ as the solution.

$$\begin{aligned} x' + y' + 4x - 3y &= 1 \\ x' - y' + x + 2y &= t \end{aligned}$$

7. Set up a system of differential equations that models the room temperature at time t for each room shown below.

A building consists of two rooms as shown below. A furnace that generates 45,000 Btu per hour heats room A. A furnace that generates 25,000 Btu per hour heats room B. The heat capacity of both rooms is $1/5$ °F per thousand Btu. The time constant for heat transfer between room A and the outside is 7 hours, between room B and the outside is 6 hours, and between room A and room B is 2 hours. The outside temperature is a constant 32°F.



8. Use Euler's method to graph the solution to $y'' + 2y' - 3y = 1 - 2t$; $y(0) = 3$; $y'(0) = -1$ on the interval $[0, 1]$ using a spacing of 0.5. Use as many lines as is necessary, but no more.

9. Use the method of annihilators to find the form of the particular solution to

$$y'''' + 6y'' - y' - 6y = t e^{-6t} + e^t + \sin t$$

10. Use the Wronskian to determine whether $\{ t^2, t^2 + 1, 2t^2 + 3 \}$ is a dependent or independent set.

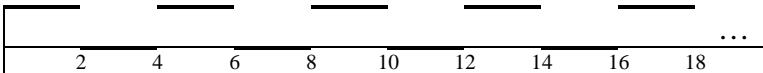
11. Prove $L\{x\}$, the Laplace transform, is a linear transformation.

12. Use the integral definition to derive the Laplace transform for 1.

13. Use the tables to find the Laplace transforms for the functions shown.

a) $L\{ e^{2t} \cos 5t \}$

b) $L\{ e^{2t} u(t-3) \}$

c) $L\{ f \}$ where f is 

14. Solve $y'''' - 8y'' + y' - 8y = 0$ Using the characteristic polynomial.

15. Use Laplace transforms to solve $y'' - 6y' + 9y = t^2 e^{3t}$; $y(0) = 2$; $y'(0) = 6$.

16. The turtles (T) in a pond eat the minnows (M) and the minnows do not eat the turtles. In the absence of turtles, the population of minnows grows at a rate equal to 40% of the current population. In the absence of minnows, the turtle population dies at a rate equal to 12% of its current population. The interaction of the two species results in the growth rate of the minnow population being reduced by 25% of the product of the population sizes, while the mortality rate of the turtle population is reduced by 10% of the product of the population sizes.

- a) Set up a system of equations that models the interaction.
- b) Determine any critical points for the system.
- c) Use your calculator to carefully graph the trajectory associated to the initial conditions $M(0) = 1$ $T(0) = 2$
- d) Explain how you know from the graph that neither population will die out.